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Optimal covering of a straight line
application to discrete convexity

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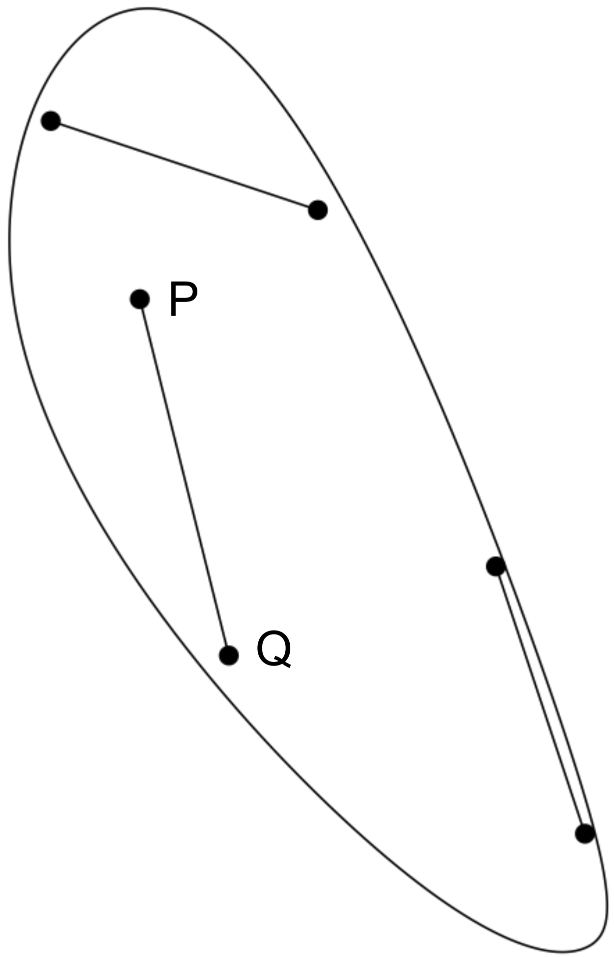
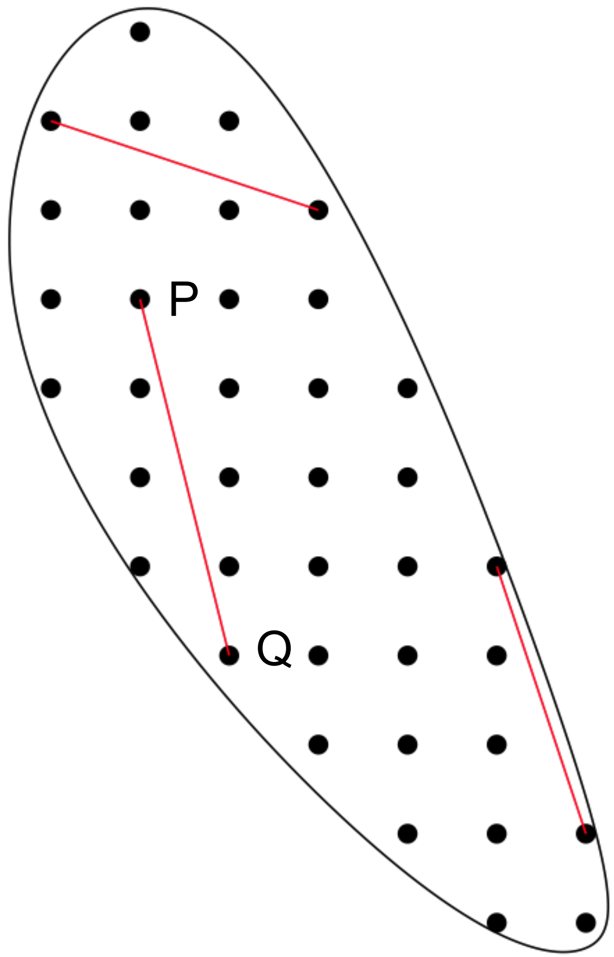
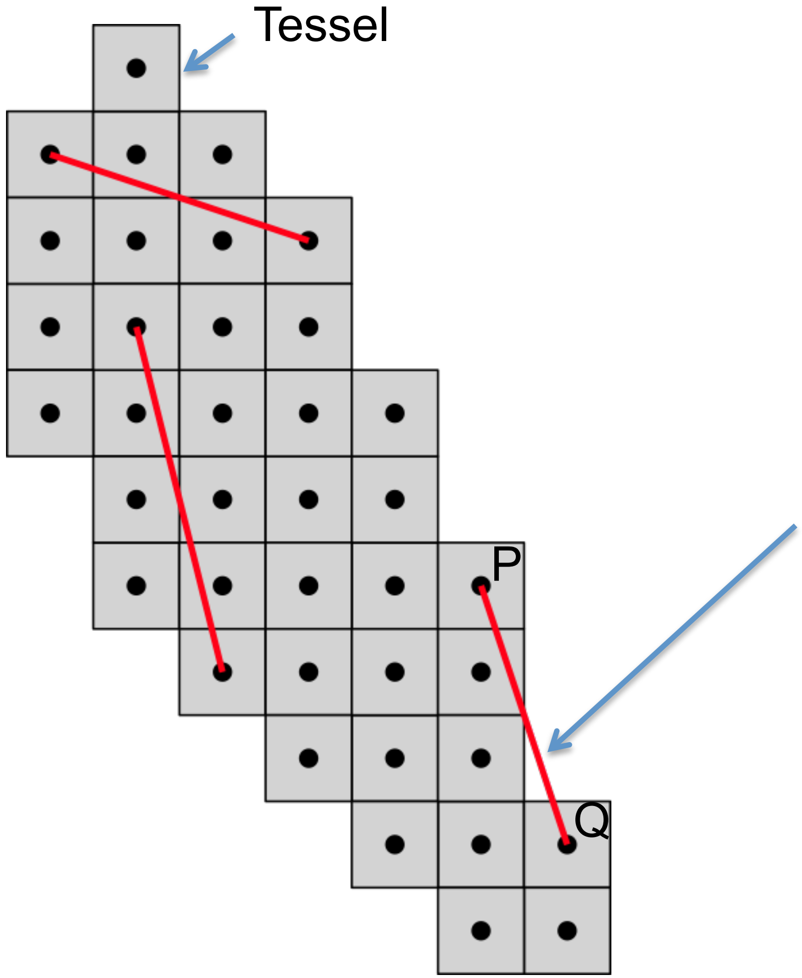
Continuous convexity



From Continuous space to discrete space



From discrete space to continuous space



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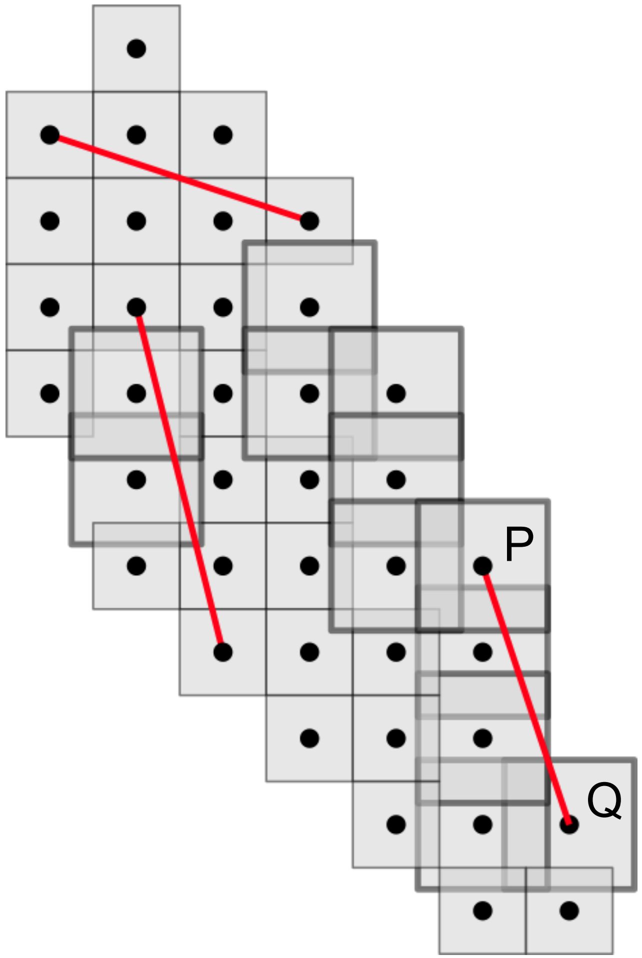
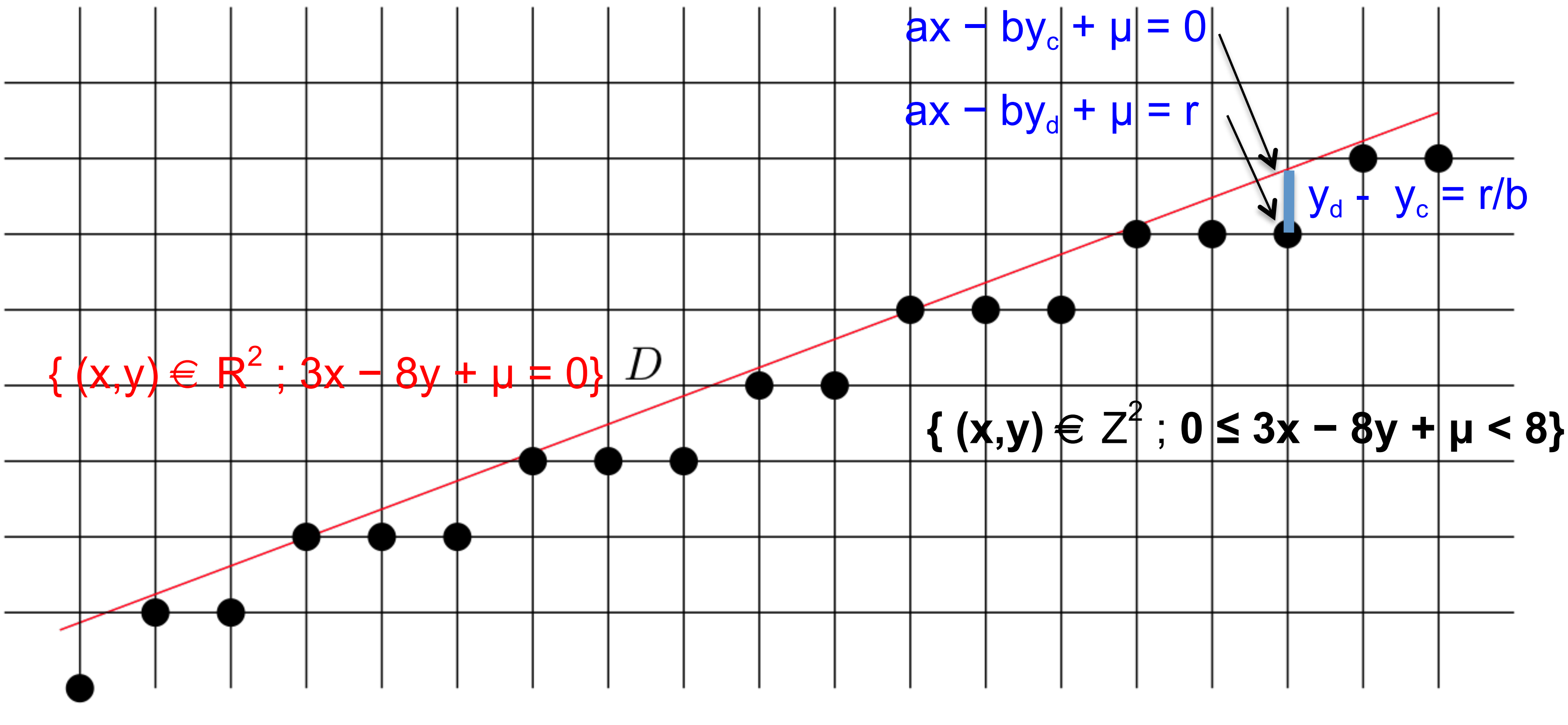
Convexity on dilated covering balls



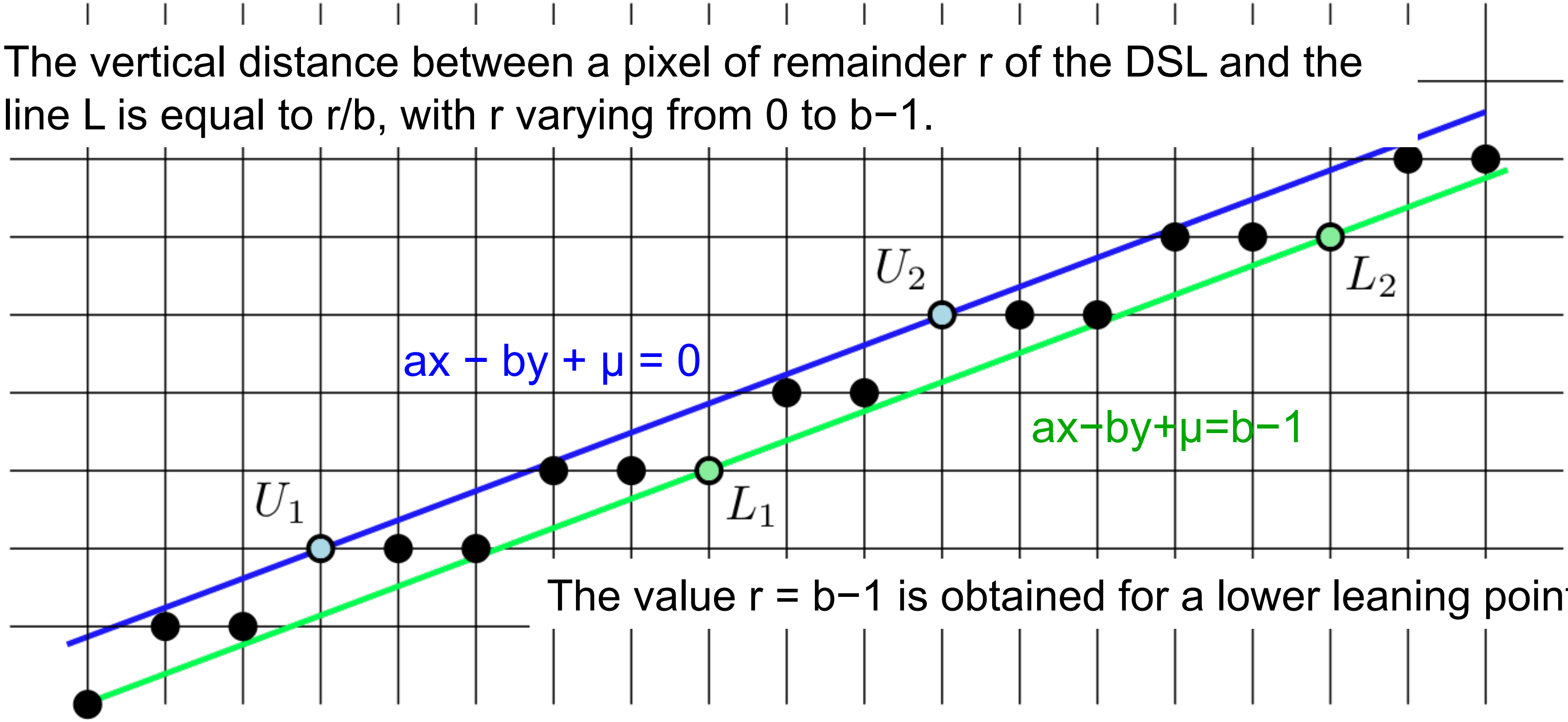
ε ? ; ½ ≤ ε < 1

Discretization of a straight line using OBQ quantization process



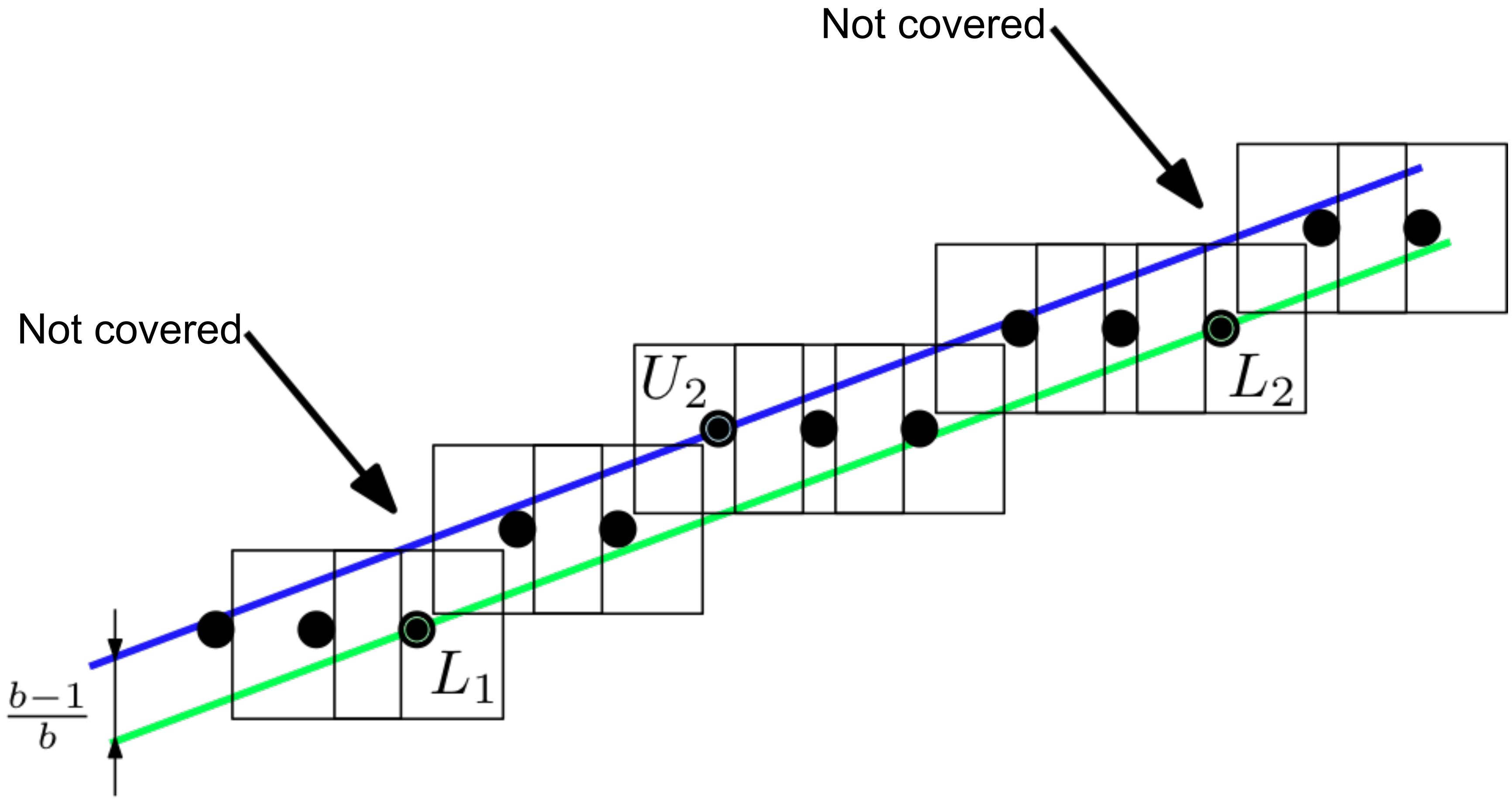
Upper and lower leaning points

The vertical distance between a pixel of remainder r of the DSL and the line L is equal to r/b, with r varying from 0 to b−1.

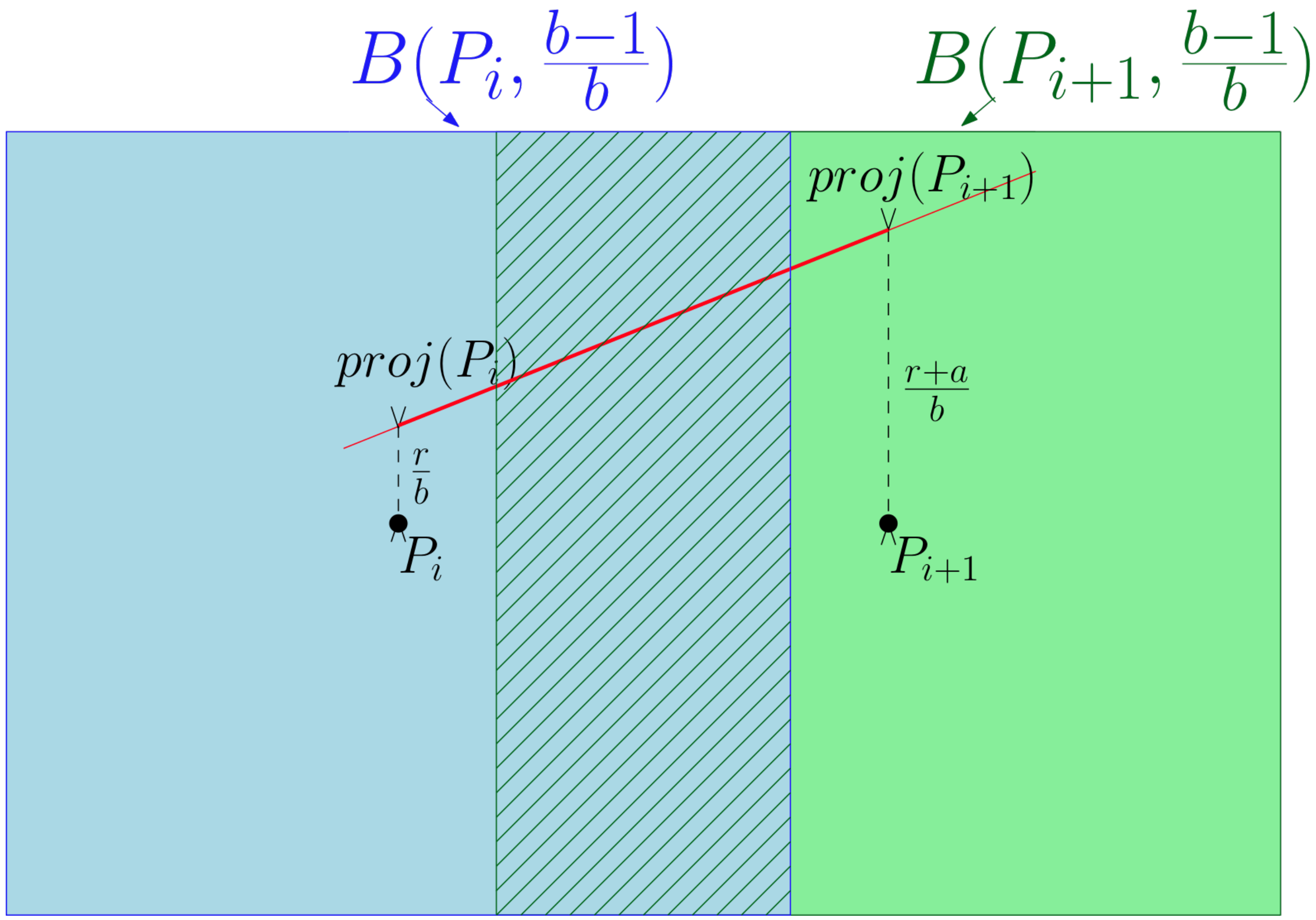


The value r = b−1 is obtained for a lower leaning point

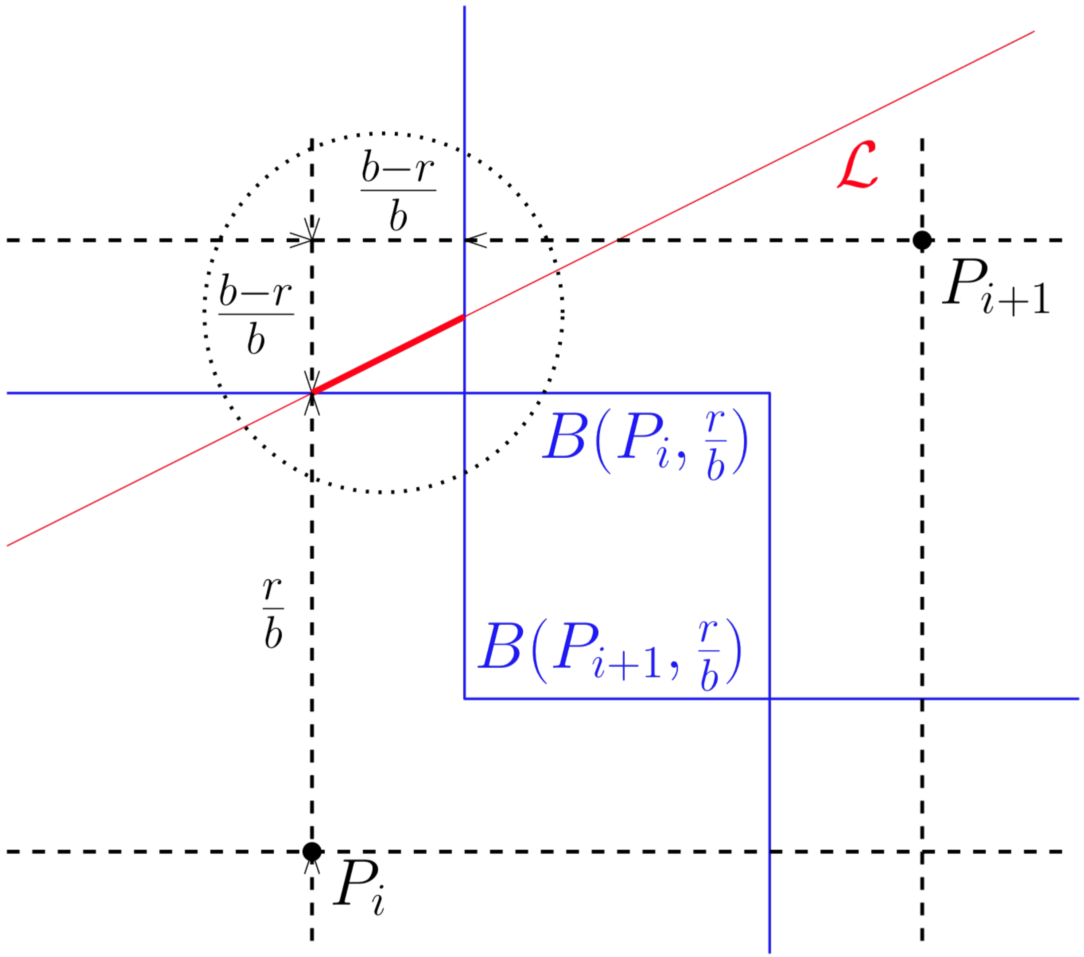
Covering by balls with radius = (b-1)/b

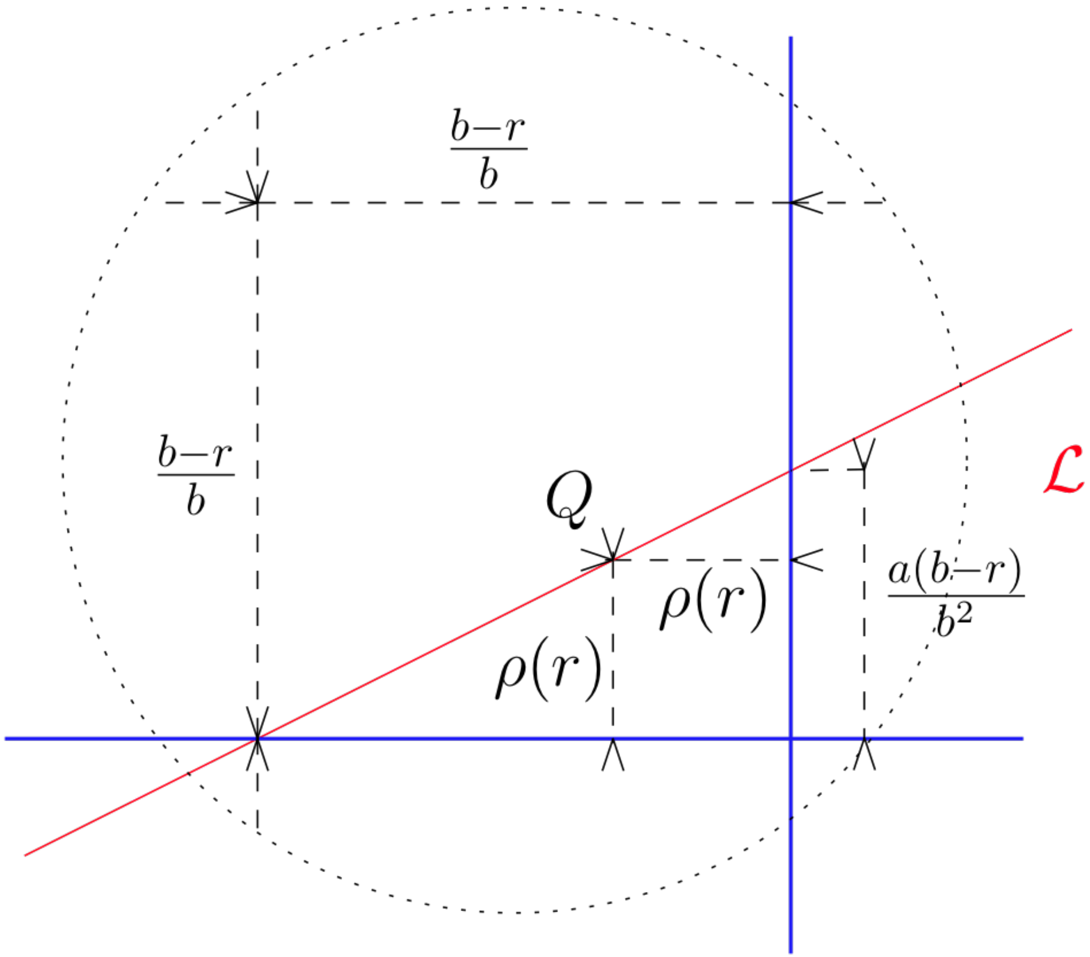


Local configuration: D = U [*proj*(P_i), *proj*(P_{i+1})]
P_i and P_{i+1} are 4-connected



Local configuration: D = U [*proj*(P_i), *proj*(P_{i+1})]
P_i and P_{i+1} are 8-connected





Using Thales Theorem
We prove:

$\rho(r)=a(b-r)/b(a+b)$

and

$\varepsilon(r)= \rho(r) + r/b$
$= (a+r) / (a+b)$

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Theorem

Let L be a straight line of equation $ax-by+\mu = 0$,

Let L its digitization with the OBQ scheme.

The union of balls $B(P_i,\varepsilon)$ centered on pixels P_i of the DSL L

with radius $\varepsilon = max(1/2, (|a|+|b|-1)/(|a|+|b|))$

covers the straight line L.

This set doesn't contain any other digital pixels excepted those of the DSL.

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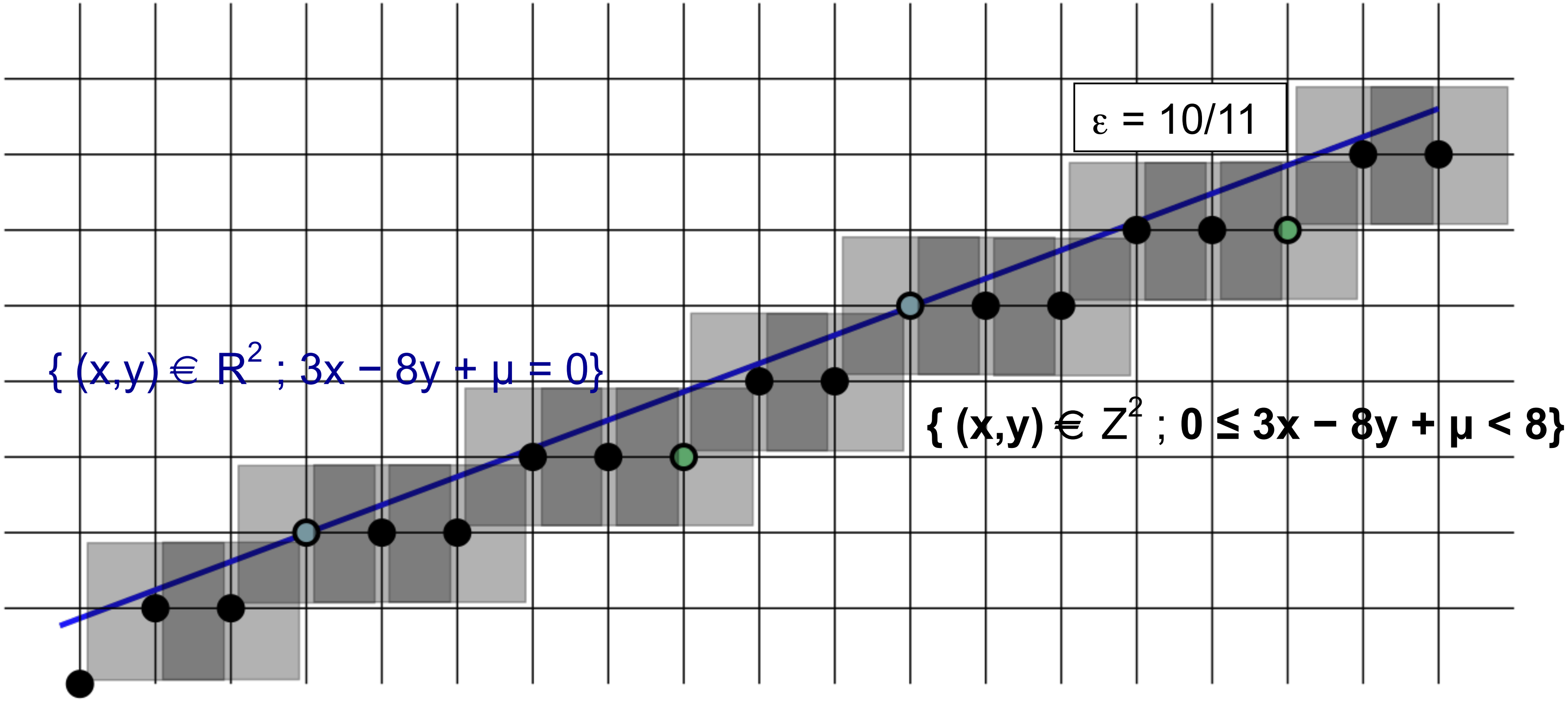
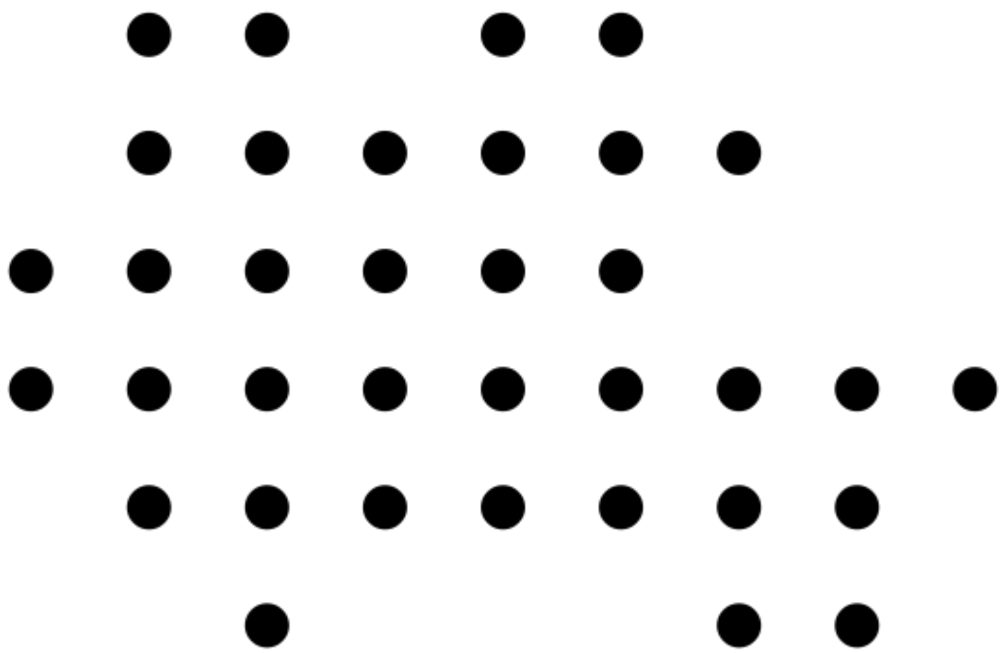
Optimal covering of the straight line

$3x - 8y + \mu = 0$



Application to discrete convexity

Initial connected component



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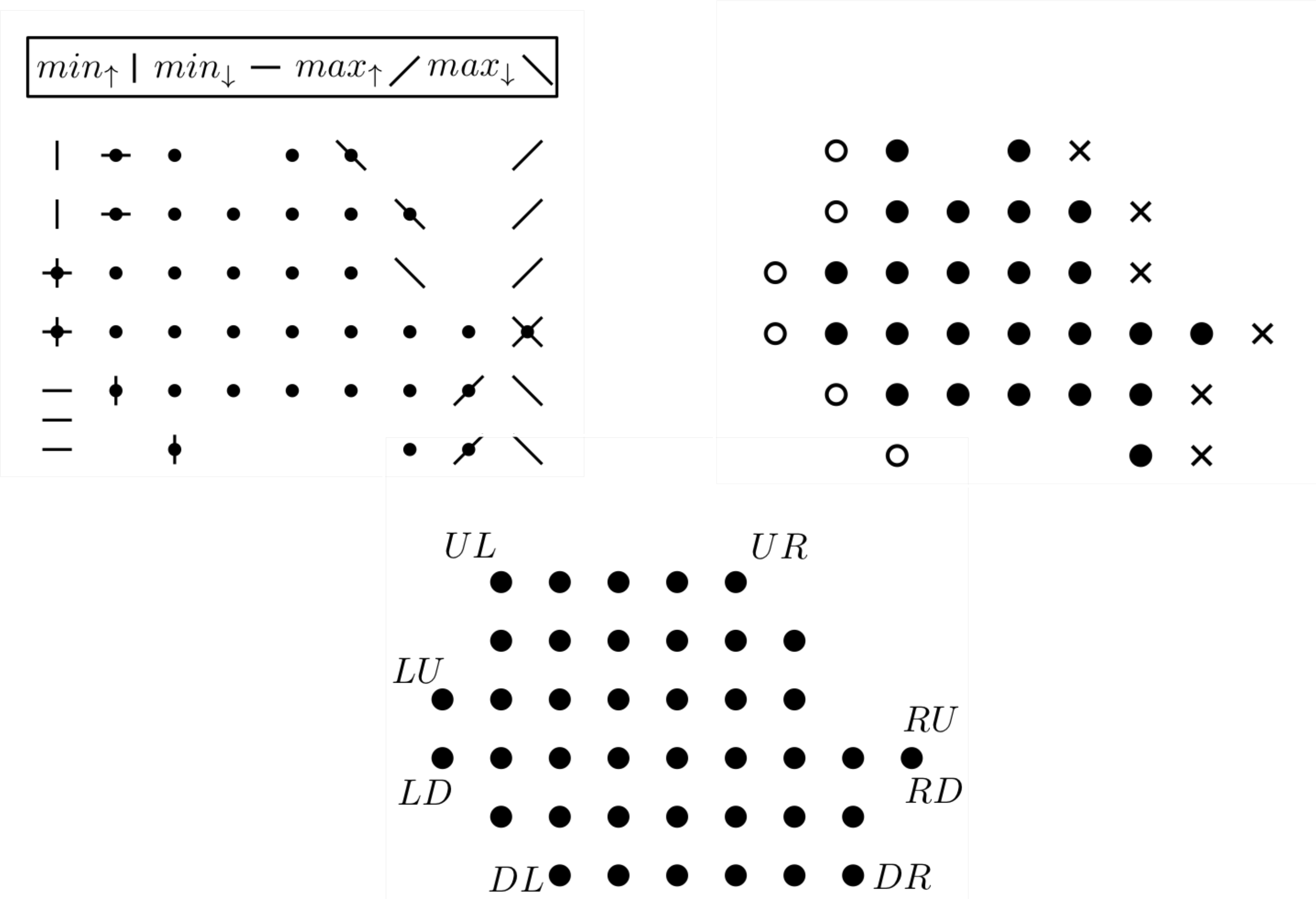
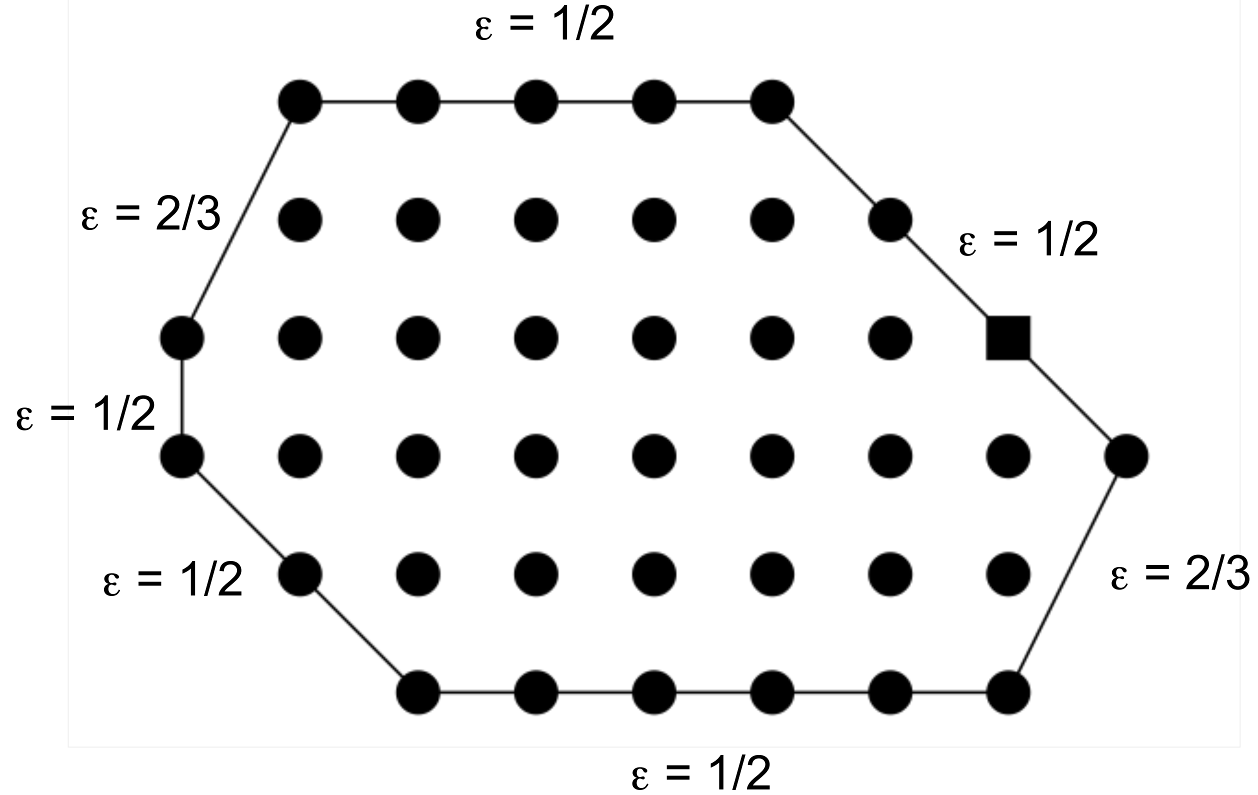
Delimitation by minimal and maximal columns



Discrete convex hull component



The discrete convex hull is ε-convex with ε = 2/3

Conclusion

- Discrete convexity is defined in perfect compatibility with continuous convexity.
- When sampling value tends to zero, discrete convexity definition is similar to continuous convexity definition.