New Structures Based on Completions

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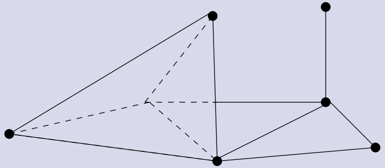
- We investigate an axiomatic approach related to combinatorial topology and simple homotopy.
- We use completions as a "language" for describing collections of objects.
- We consider objects which are simplicial complexes.



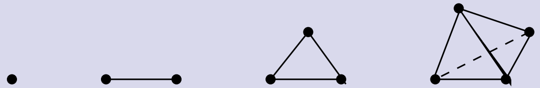
Plan of the presentation

- Simplicial complexes and completions
- Dendrites
- Dyads
- Confluence
- Relative dendrites
- Conclusion

Simplicial complexes and completions

Simplicial complexes

- Let X be a finite family composed of finite sets, X is a simplicial complex if $x \in X$ whenever $x \subseteq y$ and $y \in X$.
- We write $\mathbb{S}$ for the collection of all simplicial complexes.
- Let $X \in \mathbb{S}$. An element of X is a face of X.
- A complex $A \in \mathbb{S}$ is a cell if $A = \emptyset$ or if A has precisely one non-empty maximal face x.
- We write $\mathbb{C}$ for the collection of all cells.



Completions

- A completion may be seen as a rewriting rule which permits to derive collections of objects.
- Completions allows to formulate, in an easy way, inductive definitions.

Completions

- Let $\mathcal{K}$ be an arbitrary sub-collection of $\mathbb{S}$, $\mathcal{K}$ is a dedicated symbol (a kind of variable).
- We say that a property $\langle \mathrm{K} \rangle$ is a completion (on $\mathbb{S}$) if $\langle \mathrm{K} \rangle$ may be expressed as the following property:
  $\Longrightarrow$ If $\mathbb{F} \subseteq \mathcal{K}$, then $\mathbb{G} \subseteq \mathcal{K}$ whenever *Cond*$(\mathbb{F}, \mathbb{G})$. $\qquad \langle \mathrm{K} \rangle$
  where *Cond*$(\mathbb{F}, \mathbb{G})$ is a condition on a finite collection $\mathbb{F}$ and an arbitrary collection $\mathbb{G}$.
- *Theorem*: Let $\langle \mathrm{K} \rangle$ be a completion on $\mathbb{S}$ and let $\mathbb{X} \subseteq \mathbb{S}$. There exists, under the subset ordering, a unique minimal collection which contains $\mathbb{X}$ and which satisfies $\langle \mathrm{K} \rangle$.
- We write $\langle \mathbb{X}; \mathrm{K} \rangle$ for this unique minimal collection.
- If $\langle \mathrm{K} \rangle$ and $\langle \mathrm{Q} \rangle$ are two completions, $\langle \mathrm{K} \rangle \wedge \langle \mathrm{Q} \rangle$ is a completion, the symbol $\wedge$ standing for the logical "and". We write $\langle \mathbb{X}; \mathrm{K}, \mathrm{Q} \rangle$ for $\langle \mathbb{X}; \mathrm{K} \wedge \mathrm{Q} \rangle$.

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Example of a Completion: Connectedness

We observe that:

- A cell is connected; and
- If S and T are connected, then $S \cup T$ is connected whenever $S \cap T$ is non-empty; and
- All connected complexes may be obtained by iteratively applying the preceding rule.

We define the completion $\langle \Upsilon \rangle$ as follows:

$->$ If $S, T \in \mathcal{K}$, then $S \cup T \in \mathcal{K}$ whenever $S \cap T \neq \{\emptyset\}$. $\langle \Upsilon \rangle$

We set $\Pi = \langle \mathbb{C}; \Upsilon \rangle$, Π is precisely the collection of all simplicial complexes which are (path) connected.

We see that this completion is an alternative to the classical definition of connectedness. Furthermore it provides a constructive way for generating all connected complexes.

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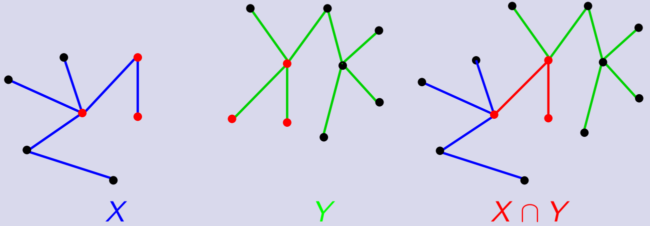
Dendrites

Motivation:

To describe a remarkable collection of acyclic complexes.

Dendrites: the basic idea

Let X and Y be two trees, and let $Z = X \cap Y$.



$X \cup Y$ is a tree whenever $X \cap Y$ is a tree

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Dendrites: the axioms

We define the completions $\langle \mathrm{D1} \rangle$ and $\langle \mathrm{D2} \rangle$ as follows:

For any $S, T \in \mathbb{S}$,

-> If $S,\ T \in \mathcal{K}$, then $S \cup T \in \mathcal{K}$ whenever $S \cap T \in \mathcal{K}$. $\langle \mathrm{D1} \rangle$

-> If $S,\ T \in \mathcal{K}$, then $S \cap T \in \mathcal{K}$ whenever $S \cup T \in \mathcal{K}$. $\langle \mathrm{D2} \rangle$

We set $\mathbb{D} = \langle \mathbb{C}; \mathrm{D1}, \mathrm{D2} \rangle$. Each element of $\mathbb{D}$ is a dendrite.

It may be shown that a complex is a dendrite if and only if it is acyclic in the sense of integral homology.

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Dyads

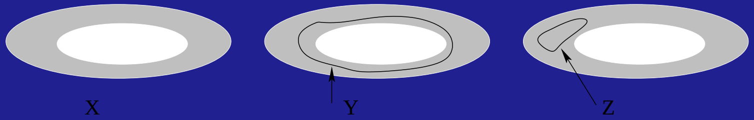
Motivation:

We want to describe collection of arbitrary complexes (complexes that are not necessarily acyclic).
It turns out that the good way to proceed was to consider couple of complexes.

Dyads

Intuitively, a dyad is a couple of complexes (X, Y), with $X \subseteq Y$, such that the cycles of X are "at the right place with respect to the ones of Y".

Three complexes X, Y, and Z, with $Y \subseteq X$ and $Z \subseteq X$:



We see that it is possible to continuously deform Y onto X, this deformation keeping Y inside X. Thus, the pair (Y, X) is a dyad. On the other hand, Z is homotopic to X, but Z is not "at the right place", therefore (Z, X) is not a dyad.

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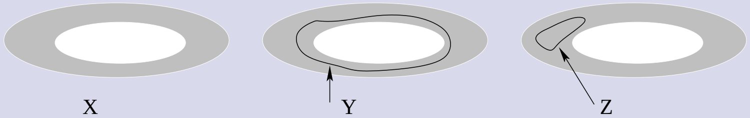

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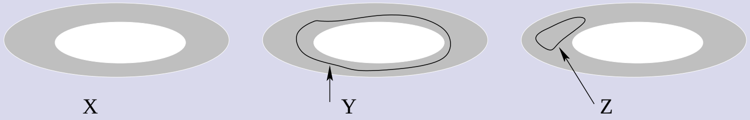

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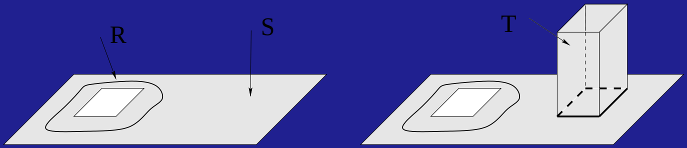
Dyads: the basic idea

- We set $\ddot{\mathbb{S}} = \{(X, Y) \mid X, Y \in \mathbb{S}, \text{ with } X \subseteq Y\}$.
- We proceed by considering completions on $\ddot{\mathbb{S}}$.
  (instead of $\mathbb{S}$)

Three complexes R, S, and T, with $R \subseteq S$:



Two objects R, S which constitute a dyad (R, S). An object T which is glued to S. The couple $(S \cap T, T)$ is a dyad, thus $(R, S \cup T)$ is also a dyad.

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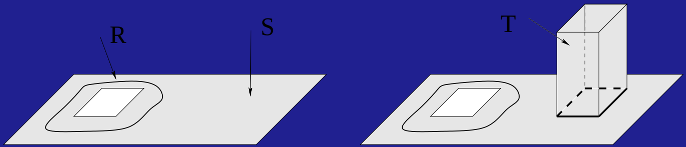

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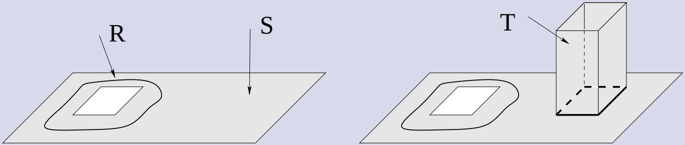

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Dyads: the axioms

We set $\ddot{\mathbb{C}} = \{(A, B) \mid A, B \in \mathbb{C}, \text{ with } A \subseteq B\}$.

We define the three completions on $\ddot{\mathbb{S}}$ as follows: For any $(R, S) \in \ddot{\mathbb{S}}$, $T \in \mathbb{S}$,

–> If (R, S) and $(S \cap T, T) \in \ddot{\mathcal{K}}$, then $(R, S \cup T) \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{X}}1 \rangle$

–> If (R, S) and $(R, S \cup T) \in \ddot{\mathcal{K}}$, then $(S \cap T, T) \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{X}}2 \rangle$

–> If $(R, S \cup T)$ and $(S \cap T, T) \in \ddot{\mathcal{K}}$, then $(R, S) \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{X}}3 \rangle$

We set $\ddot{\mathbb{X}} = \langle \ddot{\mathbb{C}}; \ddot{\mathrm{X}}1, \ddot{\mathrm{X}}2, \ddot{\mathrm{X}}3 \rangle$. Each element of $\ddot{\mathbb{X}}$ is a dyad.

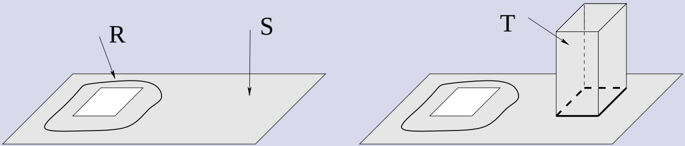
These completions constitute a set of axioms for describing couple of complexes which have "the same topology" and which are "at the right place with respect to each other".

Confluence

Motivation:
We want to describe a fundamental structure of dyads (some fundamental relationships).

Confluence: the basic idea

Three complexes R, S, and T, with $R \subseteq S$:



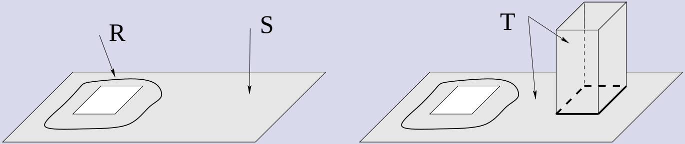
For any $(R, S) \in \ddot{\mathbb{S}}$, $T \in \mathbb{S}$,

–> If (R, S) and $(S \cap T, T) \in \ddot{\mathcal{K}}$, then $(R, S \cup T) \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{X}}1 \rangle$

Observe that S is not a subset of T

Confluence: the basic idea

Three complexes R, S, and T, with $R \subseteq S \subseteq T$:



A structural feature of dyads:

If (R, S) and (S, T) are dyads, then (R, T) is a dyad.

(Transitivity)

Confluence: the axioms

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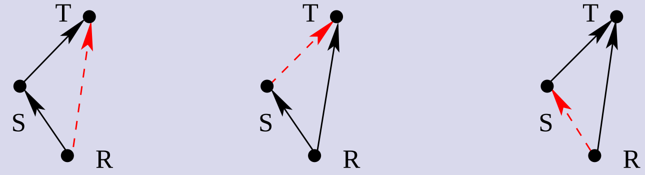

Transitivity — Upper confluence — Lower confluence

We define the three completions on $\ddot{\mathbb{S}}$ as follows:

For any $(R, S), (S, T), (R, T) \in \ddot{\mathbb{S}}$,

−> If $(R, S) \in \ddot{\mathcal{K}}$ and $(S, T) \in \ddot{\mathcal{K}}$, then $(R, T) \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{T}} \rangle$

−> If $(R, S) \in \ddot{\mathcal{K}}$ and $(R, T) \in \ddot{\mathcal{K}}$, then $(S, T) \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{U}} \rangle$

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Confluence: the confluence theorem

We define the two completions on $\ddot{\mathbb{S}}$ as follows:
For any $S, \ T \in \mathbb{S}$,

-> If $(S \cap T, T) \in \ddot{\mathcal{K}}$, then $(S, S \cup T) \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{Y}}1 \rangle$

-> If $(S, S \cup T) \in \ddot{\mathcal{K}}$, then $(S \cap T, T) \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{Y}}2 \rangle$

Theorem: We have $\ddot{\mathbb{X}} = \langle \ddot{\mathbb{C}}; \ddot{\mathrm{Y}}1, \ddot{\mathrm{Y}}2, \ddot{\mathrm{T}}, \ddot{\mathrm{U}}, \ddot{\mathrm{L}} \rangle$.

This theorem provides another way to generate the collection of all dyads. Furthermore it shows the importance of the structural relations $\langle \ddot{\mathrm{T}} \rangle$, $\langle \ddot{\mathrm{U}} \rangle$, and $\langle \ddot{\mathrm{L}} \rangle$.

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Relative dendrites

Motivation:

We want to establish a link between dyads and dendrites.

Relative dendrites: the relative dendrites theorem

We define two completions on $\ddot{\mathbb{S}}$: For any $(S, T), (S', T') \in \ddot{\mathbb{S}}$,

-> If $(S, T), (S', T'), (S \cap S', T \cap T') \in \ddot{\mathcal{K}}$, then $(S \cup S', T \cup T') \in \ddot{\mathcal{K}}$. $\langle \ddot{\mathrm{Z}}1 \rangle$

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Each element of $\langle \ddot{\mathbb{C}}^+; \ddot{\mathrm{Z}}1, \ddot{\mathrm{Z}}2 \rangle$ is called a relative dendrite.

Theorem: We have $\ddot{\mathbb{X}} = \langle \ddot{\mathbb{C}}^+; \ddot{\mathrm{Z}}1, \ddot{\mathrm{Z}}2 \rangle$.
In other words a complex is a dyad if and only if it is a relative dendrite.

This theorem provides a third way to generate the collection of all dyads. Furthermore, it allows to establish the following cancelation theorem.

Relative dendrites: the relative dendrites theorem

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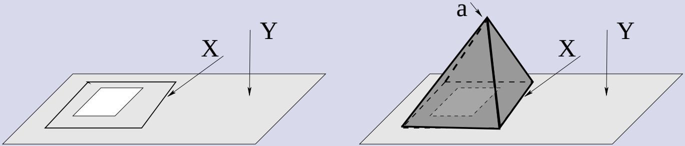
Relative dendrites: the cancelation theorem

A couple (X, Y) which is a dyad, and a cone aX:



Theorem: Let $(X, Y) \in \ddot{\mathbb{S}}$. The couple (X, Y) is a dyad if and only if $aX \cup Y$ is a dendrite.

Intuitively, this theorem asserts that, if (X, Y) is a dyad, then we cancel out all cycles of Y (*i.e.*, we obtain an acyclic complex), whenever we cancel out those of X (by the way of a cone). Furthermore, it asserts that, if we are able to cancel all cycles of Y by such a way, then (X, Y) is a dyad.

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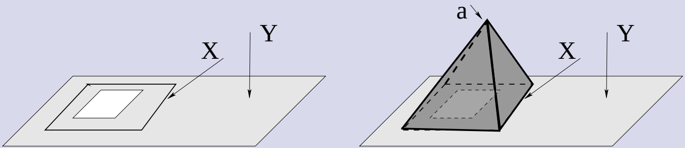

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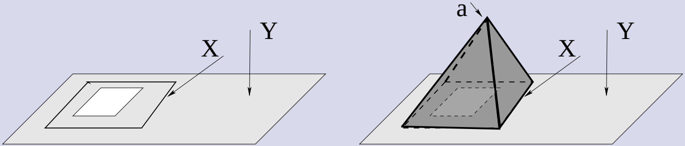
Relative dendrites: the cancelation theorem

A couple (X, Y) which is a dyad, and a cone aX:



Theorem: Let $(X, Y) \in \ddot{\mathbb{S}}$. The couple (X, Y) is a dyad if and only if $aX \cup Y$ is a dendrite.

Intuitively, this theorem asserts that, if (X, Y) is a dyad, then we cancel out all cycles of Y (*i.e.*, we obtain an acyclic complex), whenever we cancel out those of X (by the way of a cone). Furthermore, it asserts that, if we are able to cancel all cycles of Y by such a way, then (X, Y) is a dyad.

Conclusion

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- We introduced completions as a language for inductive definitions and for constructive descriptions of collections of complexes.
- We gave several theorems which show the deep links between these collections.
- These completions correspond to global topological properties of these collections.
- In the future, we will propose new completions for describing simple homotopy.

Thank you for your attention.